Virtual Session on Probability in Dynamical Systems of Physical Origin – Program

Organizers: Alex Blumenthal and Peter Nandori

June 2, 2020

We organized a special session on the above topic to be held at the AMS Spring Eastern Sectional Meeting at Tufts University this past March. Due to the COVID pandemic, this event was cancelled. We are now holding this activity online in two four-hour sessions on June 19 and June 26, 2 - 5:40 PM each day.

List of Speakers:

Kasun Fernando Akurugodage (University of Toronto)
Mark Demers (Fairfield University)
Davit Karagulyan (University of Maryland)
Maxwell Musser (Yeshiva University)
Samuel Punshon-Smith (Brown University)
Izabella Stuhl (Penn State University)
Matteo Tanzi (New York University)
Jing Zhou (U of Maryland)

Schedule:

	2:00 - 2:45	2:55 to 3:40	4:00 to 4:45	4:55 to 5:40
June 19	Mark	Kasun	Jing	Davit
June 26	Matteo	Izabella	Maxwell	Sam

Titles & Abstracts

June 19:

(2:00 - 2:45 PM) *Unique measure of maximal entropy for the finite horizon periodic Lorentz gas* by Mark Demers

Abstract: While the existence and properties of the SRB measure for the billiard map associated with a periodic Lorentz gas are well understood, there are few results regarding other types of measures for dispersing billiards. We begin by proposing a naive definition of topological entropy for the billiard map, and show that it is equivalent to several classical definitions. We then prove a variational principle for the topological entropy and proceed to construct a unique probability measure which achieves the maximum. This measure is Bernoulli and positive on open sets. An essential ingredient is a proof of the absolute continuity of the unstable foliation with respect to the measure of maximal entropy. This is joint work with Viviane Baladi.

(2:55 - 3:40 PM) *Edgeworth Expansions for Hyperbolic Dynamical Systems* by Kasun Fernando Akurugodage

Abstract: Given a dynamical system which shows hyperbolicity on a large part of phase space, one would expect it to exhibit good statistical properties like rapid decay of correlations, the Central Limit Theorem, Large Deviation Principle and etc. In this talk, I will discuss sufficient conditions for such mostly hyperbolic dynamical systems to admit Edgeworth expansions in the Central Limit Theorem. Our focus is on systems that admit a Young tower with return times with an exponentially decaying tail.

(4:00 - 4:45 PM) *Bouncing Ball in a Gravity Field* by Jing Zhou

Abstract: We study a Fermi-Ulam model where a pingpong bounces elastically against a periodically oscillating platform in a gravity field. We assume that the platform motion is piecewise C^3 with a singularity. If the second derivative of the platform motion is either always positive or always less than the negative of the gravitational constant, then the escaping orbits constitute a null set and the system is recurrent. However, under these assumptions, escaping orbits coexist with bounded orbits at arbitrarily high energy levels.

(4:55 - 5:40 PM) *Totally asymmetric dynamical walks in random environment* by Davit Karagulyan

Abstract: In this note we study dynamical random walks with internal states. We consider a one-dimensional model where a particle moves to the right ballistically, and moreover out of every three steps it moves to the right at least twice. We provide sufficient conditions for z_n to satisfy the central limit theorem.

June 26:

(2:00 - 2:45 PM) *Limit laws for random LSV maps sampled on any bounded interval in parameter space* by Matteo Tanzi

Abstract: Liverani-Saussol-Vaienti maps form a family of piecewise differentiable dynamical systems of the unit interval depending on one positive real parameter. The maps are everywhere expanding apart from a neutral fixed point and it is well known that, depending on the amount of expansion close to this point, they have either an absolutely continuous invariant measure and polynomial decay of correlations, or a unique physical measure that is singular and concentrated at the neutral point. I will present results on the composition of LSV maps whose parameters are randomly sampled so that the two contrasting behaviours are mixed. We will show that if the parameters for which there is an absolutely continuous measure are sampled with positive probability, then suitably rescaled Birkhoff averages converge to limit laws

(2:55 - 3:40 PM) *The hard-core model on 2D graphs* by Izabella Stuhl

Abstract: We will report on results about high-density Gibbs measures in the hard-core model on discrete 2D structures: a unit triangular lattice $\mathbb{A}^2$, square lattice $\mathbb{Z}^2$ and a hexagonal tiling $\mathbb{H}^2$. We describe the extreme Gibbs measures for a general value of allowed Euclidean hard-core diameter D by referring to number-theoretical properties of the (integer) number D^2. The main tool is the well-known Pirogov-Sinai theory. In particular, we address the issue of sliding arising for models on $\mathbb{Z}^2$ and $\mathbb{H}^2$.

(4:00 - 4:45 PM) *Effect of non-conservative perturbations on homoclinic and heteroclinic orbits* by Maxwell Musser

Abstract: The motivation of this work comes from astrodynamics. Consider a spacecraft traveling between the Earth and the Moon. Assume that the spacecraft follows a zero-cost orbit by coasting along the hyperbolic invariant manifolds associated to periodic orbits near the equilibrium points, at some fixed energy level. We would like to make a maneuver ? impulsive or low thrust in order to jump to the hyperbolic invariant manifold corresponding to a different energy level. Mathematically, turning on the thrusters amounts to a adding a small, non-conservative, time-dependent perturbation to the original system. Given such an explicit perturbation, we would like to estimate its effect on the orbit of the spacecraft. We study this question in the context of two simple models: the pendulum-rotator system, and the planar circular restricted three-body problem. Homoclinic/heteroclinic excursions can be described via the scattering map, which gives the future asymptotics of an orbit as a function of the past asymptotics. We add a time-dependent, non-conservative perturbation, and provide explicit formulas, in terms of convergent integrals, for the perturbed scattering map. This is based on joint work with M. Gidea and R. de la Llave.

(4:55 - 5:40 PM) *Chaotic mixing of scalars in stochastic fluid mechanics* by Samuel Punshon-Smith

Abstract: The long-time behavior of a passive scalar in an incompressible fluid has long been of interest in physics. In this talk I will discuss several recent rigorous results in this area for a passive scalar that is advected by a number of stochastic fluid models, including the stochastic Navier-Stokes equations. We will see how tools from theory of random dynamics and the ergodic and hypoelliptic theory for stochastic PDE can be used to show that the associated Lagrangian flow has a positive Lyapunov exponent. I will then discuss how to use this chaos to show that the passive scalar almost surely mixes exponentially fast, uniformly in the diffusivity, a phenomenon known in physics as chaotic mixing. Joint work with Jacob Bedrossian and Alex Blumenthal.